# MOND, ΛCDM, and Dark Matter Superfluidity: A Comparison

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I first motivate the development of the ΛCDM and MOND theories based off of observed rotation curves of galaxies, then elaborate on a theory of superfluid dark matter (SDM) recently proposed which aims to place them in a unified framework. I demonstrate that the SDM theory not only predicts the MOND acceleration, from which it can be viewed as a force mediated between superfluid phonons and baryonic matter, but also predicts a dark matter halo density profile which is cored, providing a solution to the ΛCDM cusp-core problem.

## I. MOTIVATION

It has been shown in certain instances that the dynamics of galactic structures are not well described by either the masses we perceiving them to have, or by the currently developed theories describing astrophysical phenomena. In particular, work performed by Vera Rubin in the 1970s showed that the rotational velocities of stars in galactic spirals initially increases with radial distance, but begins to flat-line at a constant value [1]. This effect is demonstrated in Fig.[1], which gives the rotational velocity for the spiral galaxy NGC 2841 as determined from its 21 cm Hydrogen radio emissions.

These observations are inconsistent with Newtonian mechanics. To see this, assume that the mass of a galaxy is concentrated near its nucleus. The velocities of stars in the galactic disks far from the nucleus of the galaxy would decrease with radius in accordance with Newton's Second Law:

$$\frac{mv^2}{r} = \frac{GMm}{r^2} \iff v = \sqrt{\frac{GM}{r}}. \tag{1}$$

If one were to orbit very close to the galactic center, the velocity would decrease with decreasing radius, as the (enclosed) mass in (1) changes. This latter fact is present in Fig.[1], but the $1/\sqrt{r}$ behavior is not present for radii beyond the galactic nucleus.

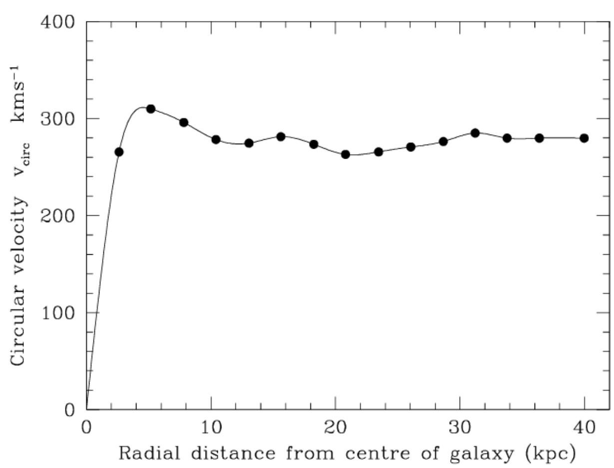



FIG. 1: Rotation velocity in $km/s$ as a function of radial distance from galactic center in $kpc$ for galaxy NGC 2841. Plot was adapted from [2], data by A. Bosma (Astron. J., 86, 1791, 1981) taken from S. M. Kent (AJ, 93, 816, 1987)

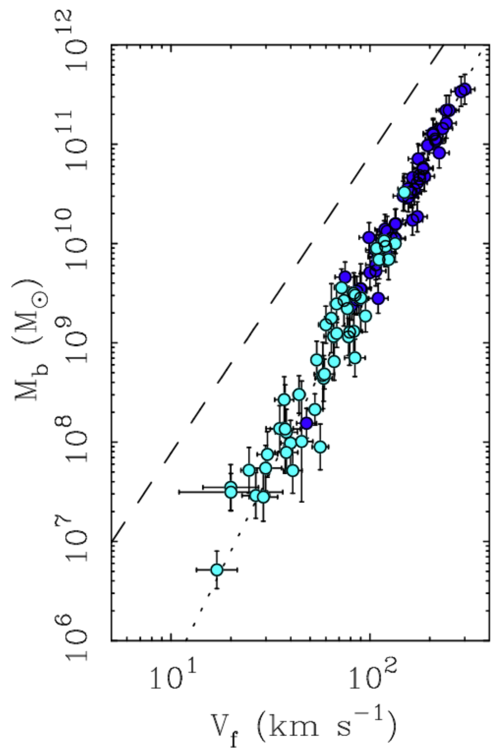



FIG. 2: Baryonic mass as a function of the outermost rotational velocity, adapted from [3]. The dotted line is the MOND prediction $\propto v_f^4$, the dashed line is the ΛCDM prediction $\propto v_f^3$, which is in less agreement to observation.

It is worth mentioning a related relation known as the baryonic-Tully-Fisher-Relation (BTFR), whose form is assumed from observations of the total baryonic mass (mass of all visible matter) and outer rotational velocity $V_f$ for various galaxies [3]:

$$M_b \propto v_f^4. \tag{2}$$

Data representing this behavior is shown in Fig.[2], and cannot be well explained by current theories of galactic phenomena.

## II. ALTERNATIVE THEORIES

These observations Fig.[1,2] point at inconsistencies in our understanding of the universe, and there are two main theories which claim to explain these findings. It could be that Newton's Laws break for the very small accelerations felt by stars sufficiently far from the galactic center. This would require a modification to Newtonian dynamics, known as *Modified Newtonian Dynamics (MOND)*, which modifies the behavior of the gravitational force at small scales [3]. There is also another leading theory, known as the Lambda cold dark matter model (ΛCDM), that assumes a different approach. It claims the existence of a "dark" matter (DM) invisible to our detection methods that couples to particles in the spirals of galaxies, leading to the flatness of rotation curves. What follows is an elaboration on both theories.

### MOND

MOND was developed by physicist Mordehai Milgram in 1983. The theory assumes that the acceleration of a star takes two forms depending on the magnitude of the Newtonian acceleration $a_N$ it would feel at its observed radius in comparison to some critical acceleration scale $a_0$. These two accelerations make up Milton's Law [4],

$$a = \begin{cases} a_N, & (a_N \gg a_0) \\ \sqrt{a_N a_0}, & (a_N \ll a_0) \end{cases} \tag{3}$$

The square root behavior is important, as for low accelerations the baryonic mass $M_b$ as a function of rotational velocity $v$ directly matches the aforementioned BTFR relation (2):

$$a_{\mathrm{MOND}}(r) = \sqrt{\frac{GM_b a_0}{r^2}} = \frac{v^2}{r} \iff M_b = \frac{v^4}{Ga_0}. \tag{4}$$

So the $a_0$ not only provides a notion of "large" or "small" accelerations, but also sets the relationship between $M_b$ and $v$. The best-fitting value for $a_0$ is, conveniently, on the order of the Hubble constant $a_0 \sim H_0 \sim 1.2 \times 10^{-8}$ cm/s$^2$ [4]. Hence, the validity of (4) is for $a \ll 10^{-8}$ cm/s$^2$.

It's quite clear that MOND is a non-relativistic theory, particularly evident through my calculation methods (4). MOND is blind to the notion of gravity being a manifestation of spacetime curvature, and hence will not provide an accurate prediction of the perihelion precession of mercury, or predict the existence of black holes. It was devised as a simple model to gauge away the necessity of explanations in the form of dark matter or energy from the ΛCDM theory. There does happen to be a relativistic generalization of MOND, known as tensor-vector-scalar (TeVeS) theory, which gives good predictions for gravitational lensing and structure formation but fails when applied to the anisotropy of the CMB [5].

Another prominent issue with the MOND theory is the external field effect (EFE). In a many-body system, the linearity of Newtonian acceleration allows one to describe the dynamics of a collection of particles by their center of mass: the large system has no influence on the subsystem, and we can decouple the effects of the environment [5]. However, since (4) is non-linear in acceleration, this de-coupling cannot occur and could in principle result in non-local effects, namely, a violation of the strong equivalence principle (SEP) in which the dynamics of a particle are sensitive to the strength of an external gravitational field. Milgram has faith in his theory and proposes that this can be viewed as a prediction of the violation of the SEP [6], which is backed up by recent work [7] showing statistical evidence of the violation of the SEP for weak fields around rotationally supported galaxies.

### ΛCDM

There are two key components to ΛCDM. It firstly proposes the presence of a cosmological constant Λ that augments Einstein's field equations,

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi G T_{\mu\nu} \tag{5}$$

consistent with an interpretation as a non-zero vacuum energy, or "dark" energy that forms a perfect fluid in spacetime. The associated energy density is constant as the universe expands, $\rho_\Lambda \propto a^0$, and its equation of state is that of negative pressure: $P_\Lambda \propto -\rho_\Lambda$. Since the energy density for matter and radiation both decrease as the universe expands [8], over time the dark energy component will take over, and its negative pressure helps to explain the observed acceleration of the expansion of the universe.

The second component of the theory is cold dark matter (CDM). Candidates for this type of non-baryonic matter must be cold, and couple only with gravitational fields. By "cold", I mean that they are non-relativistic, allowing them to form structures upon decoupling from photons in the early universe. Since they couple only with gravitational fields, they can be described as dissipation-less in the sense that they possess no mechanism of lowering their energy through EM radiation [3]. This fact means the structures they form are not as robust as that of baryonic matter; CDM forms gravitational wells which provide a foundation to the large scale structure formed by baryonic matter, and this foundation is what we denote as the DM halos which surround galactic structures, giving rise to observations consistent with Figs.[1,2].

ΛCDM is a largely successful theory, giving explanation to larger scale structures of spacetime, but it is not without its pitfalls. For instance, a BTFR of the form (2) is not predicted by ΛCDM, a result of the fact that the acceleration scale $a_0$ is absent.

### A Bridge to DM Superfluidity

The acceleration scale $a_0$ can be viewed either as a numerical coincidence, or as a fundamental fact of nature, but its presence in both the $\Lambda$ and CDM sectors provides evidence against the notion of treating dark matter and dark energy independently [3]. This fact, among others, has led to the creation of theories which attempt to unify the two, one of which being a theory of dark matter superfluidity proposed by L. Berezhiani and J. Khoury [4]. I will first provide a brief description of superfluidity in the context of condensed matter systems, then describe the theory proposed by Berezhiani & Khoury, making connections between both MOND and $\Lambda$CDM along the way.

## III. DARK MATTER SUPERFLUIDITY

### Aside on Superfluidity

Superfluidity is quite a peculiar phase of matter, and is fundamentally connected with the concept of Bose-Einstein Condensation (BEC). BEC occurs in certain bosonic systems cooled near absolute zero, from which a portion of the particles condense into the same quantum state. This manifests quantum properties on a macroscopic scale, one of which being superfluidity [9]. An example is Helium-4, which when cooled beyond 2 K has the uncanny ability to climb the walls of its container, or leak directly through the container via minuscule capillaries originally capable of containing the liquid at higher temperatures. These observations are consistent with zero-viscocity, dissipation-less motion.

Such properties can be viewed as a manifestation of spontaneous symmetry breaking (SSB). As the system is cooled beyond the critical BEC temperature, the coherence of the quantum state means a particular phase for the macroscopic wavefunction is picked, breaking the conservation of particle number, $\psi \to |\psi|e^{i\phi}$ [9]. The phase $\phi$ is the Goldstone mode (phonon) for this broken U(1) symmetry, and its fluctuations can couple to other degrees of freedom in the system, giving rise to interesting properties. The effective field theory of these fluctuations can be described by

$$\mathcal{L} = P(X), \qquad X = \dot{\phi} - \frac{1}{2}(\nabla\phi)^2 \tag{6}$$

where $P(X)$ is a function describing the equation of state of the fluid.

### The Novel Idea

Berezhiani & Khoury hypothesize that DM particles BEC condense into a superfluid with coherence length on the order of galactic scales [4]. By appealing to superfluidity, the dissipation-less nature of CDM has a natural explanation, and the effects of DM and MOND can be described as different phases of the superfluid. In particular, the macroscopic behavior of the DM condensate is less like that of individual, collision-less particles and more akin to that of collective excitations: phonons [4]. The effective field theory of these DM phonons has the same structure as (7), albeit with the introduction of a Newtonian gravitational potential,

$$\mathcal{L} = P(X), \qquad X = \dot{\phi} - m\Phi - \frac{1}{2}(\nabla\phi)^2 \tag{7}$$

### Comparison with MOND Acceleration

In order to agree with MOND accelerations (4) in rotation curves, it must be that the DM phonons possess an equation of state

$$P(X) \sim \Lambda X\sqrt{|X|}, \tag{8}$$

where $\Lambda \sim$ meV is chosen to give the critical acceleration $a_0 \sim 10^{-8}$ within the MOND framework. The fractional power (8) is intriguing, but not uncommon in the theory of cold atom systems, from which this theory has parallels with. Namely, the onset of superfluid DM condensation occurs when the DM particles begin self-interacting, around $T \sim$ mK [4], which is similar to that of cold atom systems. In cold atoms systems, the atoms are trapped by magnetic fields, but here the cold DM particles are trapped by gravitational fields.

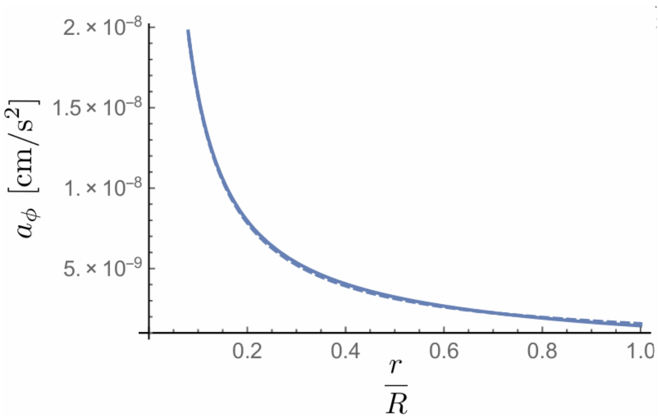
The MOND acceleration is physically achievable through the introduction of an interaction term $\mathcal{L}_{\text{int}} \propto -\alpha\phi\rho_b$, $\alpha = \text{const.}$ that couples the phonons to baryonic matter, and represents the only other input to the theory. Hence, a particle orbiting a galaxy experiences the usual gravitational force, as well as a phonon-mediated force, and this phonon-mediated force is conjectured to be that which gives rise to the flat rotation curves Fig.[1].

Following a detailed derivation [4], at finite temperature the phonon-mediated acceleration predicted from the superfluid framework $a_\phi \propto \alpha\nabla\phi(r) \equiv \alpha\phi'(r)$ for a galaxy with baryonic mass $M_b(r)$ can be determined from the equation

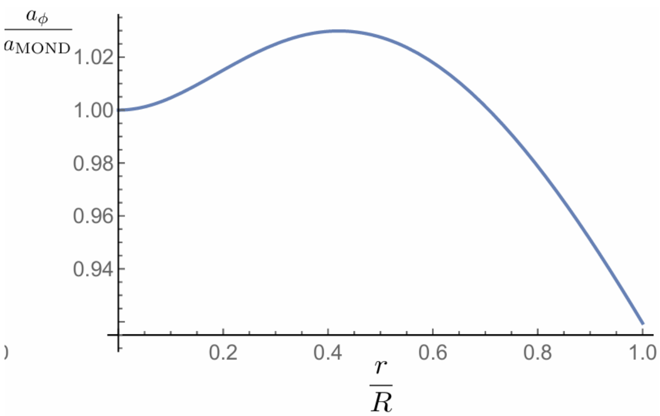
$$\frac{a_\phi(r)^2 + 2m\alpha^2\left(\frac{2\beta}{3} - 1\right)(\mu - m\Phi)}{\sqrt{a_\phi(r)^2 + 2m\alpha^2(\beta - 1)(\mu - m\Phi)}}a_\phi(r) = \frac{\alpha^3 M_b(r)}{8\pi\Lambda r^2}, \tag{9}$$

where $\mu$ is the chemical potential of the DM superfluid, and $\beta \geq 3/2$ is a dimensionless parameter characterized by the deviation of condensation temperature to critical BEC temperature [4].

Numerical solutions to (9) are plotted in Fig.[3], as well as the ratio $a_\phi(r)/a_{\text{MOND}}(r)$ for $a_{\text{MOND}}$ given by (4). It is evident from Fig.[3] that the phonon-mediated acceleration closely matches the MOND acceleration (4), but this is hardly a consequence. The equation of state (8) for the superfluid DM was chosen in a manner that mimics MONDian behavior.




(a) Plot of $a_\phi(r)$ (9) as a function of $r/R$, where $R$ is the radius of the DM halo, adapted from [4]. The parameters are chosen for a MW-type galaxy: $M_b = 3 \times 10^{11}$ $\mathrm{M}_\odot$, as well as $m = 0.6$ eV, $\Lambda = 0.2$ MeV, and $\beta = 2$. The dashed line is the MOND prediction (4), which is in great agreement.




(b) Ratio of $a_\phi(r)$ to the MOND acceleration $a_{\mathrm{MOND}}$ (4), which is within a few percent throughout the entire halo. Plot is adapted from [4].

FIG. 3

**Solutions to Aspects of MOND/ΛCDM**

It is known that the MOND theory has issues in the fact that $a_\phi$ gives large corrections to Newtonian gravity, and the way to avoid them requires fine tuning or implementing higher-order derivatives, complicating the theory [4]. The superfluid DM theory has a simple solution to this issue. Note that if the local superfluid velocity $v_s = |\nabla\phi|/m$ does not exceed the BEC critical velocity, $v_s \ll v_c$, the phonon gradient is small and the BEC remains coherent. If not, then the condensate gets replaced by a phase of ordinary DM.

By estimating values for $v_s$ and $v_c$, it can be shown that on solar system scales the phonon gradient of the sun is large enough to violate the coherence condition $v_s \ll v_c$, causing the condensate to lose coherence [4]. This makes perfect sense, the MOND acceleration (4) is only valid for small accelerations, and the accelerations on a solar system scale are usually quite large. Hence, there is no longer any issue with local tests of gravity present in the MOND theory, and the resulting DM particle phase can in principle be directly-detected through axion-like particle searches [4].

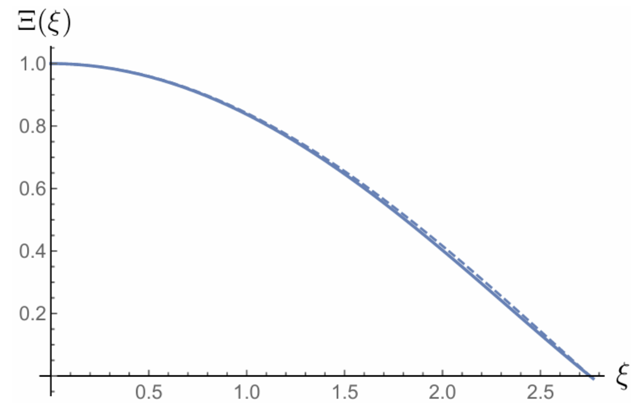



FIG. 4: Numerical computation of the DM halo density profile for dimensionless parameters (11). The density is naturally cored, and vanishes at a critical radius $\xi \sim 2.7$. Plot adapted from [4].

In the case of the ΛCDM theory, the equation of state (8) predicts the relation

$$P \sim \rho^3, \tag{10}$$

between the pressure $P$ and energy density $\rho$, which has consequences on the density profile of DM halos. Assuming hydrostatic equilibrium, the equation for the density profile $\rho(r)$ can be derived [4], with numerical solutions plotted in Fig.[4] for dimensionless parameters

$$\Xi^{1/2}(r) = \rho(r)/\rho_0, \qquad \xi = \left(\frac{32\pi G\Lambda^2 m^6}{\rho_0}\right)^{1/2} r, \tag{11}$$

where $\rho_0 = \rho(0)$ is the critical density.

Fig.[4] solves an important issue in the ΛCDM framework known as the *core-cusp problem* [10]. Observations of galaxies dominated by dark matter indicate that the central regions have cores where the density is approximately constant. This is in conflict with the ΛCDM prediction of a "cusp"-like behavior $\rho \sim 1/r$ for small $r$. The density predicted by the superfluid DM theory is naturally cored Fig.[4], as for small $r$ the density is approximately constant $\Xi \sim \rho^2 \sim 1$, and there is no longer such a discrepancy between theory and observation.

## IV. CONCLUDING REMARKS

Multiple theories have been proposed to explain the inconsistencies between fundamental theory and observational data of galactic structures, namely that of flat rotation curves Fig.[1]. Two common theories, modified

Newtonian dynamics (MOND) and ΛCDM, provide explanations, either through a modified Newtonian force or by the presence of invisible "dark" matter, but a common link between the two has not been established. By proposing that dark matter consists of a superfluid, it is possible to unite the MOND and ΛCDM theories, in which case the MOND force can be viewed as a force mediated between superfluid phonon excitations and baryonic matter.

The theory not only provides an accurate prediction for the MOND force, explaining the flatness of galactic rotation curves, but also alleviates issues with the MOND and ΛCDM theories, namely local tests of gravity in the MOND framework and the cusp-core problem of ΛCDM. These successes provide validity to the superfluid dark matter theory, but with its development being recent and quite speculative, further predictions of the theory have yet to be observed. Nonetheless, the theory of dark matter superfluidity is an intriguing proposal which can be viewed as a stepping stone in our understanding of the universe on larger scales.

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